RIMS Workshop 2022
Analytic Number Theory and Related Topics

Organizers: Yoshinori Yamasaki (Ehime University)
Yu Yasufuku (Nihon University)

Date: October 11 (Tue) 09:50 – October 14 (Fri) 16:40, 2022
Place: Room 420, RIMS, Kyoto University, Japan
Format: Hybrid (On-site and via Zoom Meeting)

Abstracts

October 11 (Tue)

10:00 – 11:00 **Takashi Taniguchi (Kobe University)**
"Improved error estimates for counting cubic fields"
(joint work with Frank Thorne and Manjul Bhargava)

In this talk, we improve the error term estimate in the counting function for cubic fields. We also prove a version of our theorem that counts cubic fields satisfying certain local conditions, where we largely improve the dependence of the error term on the conditions.

11:20 – 12:00 **Kota Saito (University of Tsukuba)**
"A system of certain linear Diophantine equations on analogs of squares"
(joint work with Yuya Kanado)

This study investigates the existence of tuples (k, ℓ, m) of integers such that all of k, ℓ, m, $k+\ell$, $\ell+m$, $m+k$, $k+\ell+m$ belong to $S(\alpha)$, where $S(\alpha)$ is the set of all integers of the form $\lfloor \alpha n^2 \rfloor$ for $n \geq \alpha^{-1/2}$ and $\lfloor x \rfloor$ denotes the integer part of x. We show that $T(\alpha)$, the set of all such tuples, is infinite for all $\alpha \in (0,1) \cap \mathbb{Q}$ and for almost all $\alpha \in (0,1)$ in the sense of the Lebesgue measure. Furthermore, we show that if there exists $\alpha > 0$ such that $T(\alpha)$ is finite, then there is no perfect Euler brick. This is joint work with Yuya Kanado (Nagoya University).

13:40 – 14:40 **Jaehyun Cho (Ulsan National Institute of Science and Technology)**
"On analytic ranks of elliptic curves with prescribed torsion"
(joint work with Keunyoung Jeong)

We show that the average analytic rank of elliptic curves with prescribed torsion G is bounded for every torsion group G under GRH for elliptic curve L-functions and in addition some moment conditions for $G = \mathbb{Z}/n\mathbb{Z}$, $n = 7, 8, 9, 10, 12$ and $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2m\mathbb{Z}$, $m = 3, 4$.

15:00 – 15:30 **Hideki Matsumura[Z] (Keio University)**
"Elliptic analogue of irregular prime numbers for the p^n-division fields of the curves $y^2 = x^3 - (s^4 + t^2)x$"
(joint work with Naoto Dainobu and Yoshinosuke Hirakawa)

A prime number p is said to be irregular if it divides the class number of the p-th cyclotomic field $\mathbb{Q}(\zeta_p) = \mathbb{Q}(\mathbb{G}_m[p])$. In this talk, we study its elliptic analogue for the division fields of an elliptic curve. More precisely, we study the p-divisibility of the class number of the p^n-division field $\mathbb{Q}(E[p^n])$ of an elliptic curve E. On the p-part of the ideal class group of $\mathbb{Q}(E[p^n])$, several lower bounds are obtained in the literature including recent works by Sairaiji-Yamauchi,

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[Z] Talk by Zoom online

Hiranouchi, Ohshita, and Prasad-Shekhar. Their works imply that the class number of the p^n-division field is often divisible by a certain power of p if the Mordell-Weil rank of E is larger than 1. In this talk, we consider a family $\{E_{s,t}\}$ of elliptic curves defined by $y^2 = x^3 - (s^4+t^2)x$ with integers s, t. Then, for every fixed $p \geq 5$ and n, we can prove that the above family contains a certain infinite subfamily having the following two novel properties: Their Mordell-Weil ranks are all 1 and the class numbers of their p^n-division fields are all divisible by p^{2n}. In our construction, we use the Friedlander-Iwaniec theorem for prime numbers of the form s^4+t^2. This is a joint work with Naoto Dainobu and Yoshinosuke Hirakawa (arXiv:2205.08946).

15:50 – 16:20 **Yutaro Matsuno (Waseda University)**
"On generalization of Hurwitz zeta functions on algebraic number fields and Euclidean minima"

Let K be an algebraic number field with the integral ring $\mathfrak{o}_K$. For $\alpha \in K$, the number $M_K(\alpha) = \min_{\theta \in \mathfrak{o}_K} |N_{K/\mathbb{Q}}(\alpha + \theta)|$ is called the Euclidean minima of α and $M(K) = \sup_{\alpha \in K} M_K(\alpha)$ is called Euclidean minima of K. It is known that $M(K)$ is related to norm Euclidity of K. In this lecture, we shall define generalized ideal classes and generalized Hurwitz zeta functions and show their functional equation, their values of non-negative points on totally real fields (especially on real quadratic fields) and the relation to Euclidean minima.

October 12 (Wed)

9:30 – 10:30 **Dorian Goldfeld[Z] (Columbia University)**
"Eisenstein series for $\mathrm{SL}(n, \mathbb{Z})$"
(joint work with Eric Stade and Michael Woodbury)

In this talk I will explain the construction (in elementary classical terms) of the most general Langlands Eisenstein series which are automorphic for the group $\mathrm{SL}(n, \mathbb{Z})$ with $n > 1$. The non constant Fourier coefficients of these Eisenstein series will be explicitly presented as generalized divisor sums multiplied by products of completed (including Gamma factors and powers of π) Rankin-Selberg L-functions and adjoint L-functions. This is joint work with Eric Stade and Michael Woodbury.

10:50 – 11:30 **Yuta Suzuki (Rikkyo University)**
"An average Manin conjecture with weak approximation on Fano hypersurfaces"
(joint work with Yohsuke Matsuzawa)

Consider the set of hypersurfaces $\mathbb{V}_{d,n}$ of degree $d \geq 2$ in $\mathbb{P}^n$ defined over $\mathbb{Q}$. For a given $V \in \mathbb{V}_{d,n}$, we are interested in the number $N_V(B)$ of rational points on V up to height B. There is an expected asymptotic formula for $N_V(B)$ of general $V \in \mathbb{V}_{d,n}$ in the Fano range $n \geq d$, which is a special case of the Manin conjecture. In his celebrated work, Birch applied the circle method to $N_V(B)$ and obtained the expected asymptotic formula, but unfortunately, the result has a strong restriction $n \geq (d-1)2^d$. Recently, le Boudec proved instead an asymptotic formula for the direct average of $N_V(B)$ over V in the full (!) Fano range $n \geq d \geq 2$ except $(n, d) = (2, 2)$, which can be thought of as the averaged Manin conjecture. In this talk, we combine this problem with the weak approximation conjecture (a generalization of the Chinese remainder theorem to V) and report the resulting average Manin conjecture with weak approximation (i.e. with congruence condition). We require uniformity over the approximation, which introduces new difficulties to the problem. This is a joint work in progress with Yohsuke Matsuzawa (Rikkyo University).

13:10 – 13:50 **Tatsushi Tanaka (Kyoto Sangyo University)**
"On interpolated multiple L-values"

[Z] Talk by Zoom online

(joint work with Shin-ya Ito and Noriko Wakabayashi)

Interpolation method for multiple zeta values invented by Shuji Yamamoto (University of Tokyo) is applied for multiple L-values. Algebraic setup, special values, and relations for interpolated multiple L-values will be presented. This is a joint work with Shin-ya Ito (former graduate student of Kyoto Sangyo University) and Noriko Wakabayashi (Osaka Electro-Communication University).

14:10 – 14:50 **Shin-ya Kadota (National Institute of Technology (KOSEN), Niihama College)**

"On a unified double zeta function of Mordell-Tornheim type"
(joint work with Takuya Okamoto, Masataka Ono and Koji Tasaka)

Komori introduced a unified multiple zeta function and gave some results on analytic properties and special values at nonpositive integer points. In this talk, we introduce a unified multiple zeta function of Mordell-Tornheim type and give some results on analytic properties and special values at nonpositive integer points.

15:10 – 15:40 **Shintaro Murakami (Hirosaki University)**

"Linear independence of certain gap series"
(joint work with Yohei Tachiya)

In 2019, V. Kumar obtained linear independence results for certain gap series with monomial exponents under some conditions. In this talk, we give some generalizations of Kumar's results by removing his conditions. Our methods are based on the irrationality proofs of S. Chcowla (1947) and P. Erdős (1948), and the result of K. Mahler (1953) on finiteness of solutions of certain diophantine equation. This is joint work with Yohei Tachiya (Hirosaki University).

16:00 – 16:40 **Hajime Kaneko (University of Tsukuba)**

"New relation for the coefficients of cyclotomic polynomials"
(joint work with Shigeki Akiyama)

Cyclotomic polynomials, which are the minimal polynomials of primitive roots of unity, are classical objects of number theory. The coefficients of cyclotomic polynomials have attractive properties. In this talk we introduce new relation for the coefficients of cyclotomic polynomials. As a corollary, we give divisibility relation between the values of the derivatives of cyclotomic polynomials and the values related to Euler's totient function. This is a joint work with Shigeki Akiyama.

October 13 (Thu)

9:30 – 10:30 **Simon Marshall (University of Wisconsin-Madison)**

"Large values of eigenfunctions on hyperbolic manifolds"
(joint work with Farrell Brumley)

It is a folklore conjecture that the sup norm of a Laplace eigenfunction on a hyperbolic surface grows more slowly than any positive power of the eigenvalue. In dimensions three and higher, this was shown to be false by Iwaniec-Sarnak and Donnelly. I will present joint work with Farrell Brumley that strengthens these results, and extends them to locally symmetric spaces associated to $\mathrm{SO}(p, q)$.

10:50 – 11:30 **Yuya Murakami (Tohoku University)**

"An asymptotic formula for false theta functions and quantum invariants of plumbed homology spheres"

Recently, modular forms have been applied to the study of asymptotic of quantum invariants

in 3-dimensional topology. In this talk, I will prove the Gukov-Pei-Putrov-Vafa conjecture for conditional plumbed homology spheres. A key of my proof is a new technique for asymptotic expansions, which enables us to compare asymptotic expansions of rational functions and false theta functions.

13:10 – 13:50 **Shingo Sugiyama (Nihon University)**

"Weighted one-level density for Dirichlet L-functions"
(joint work with Ade Irma Suriajaya)

At the end of the 20th century, Katz and Sarnak suggested that low-lying zeros of L-functions in a family should be distributed like eigenvalues of random matrices in a compact matrix group. Inspired by Kowalski, Saha and Tsimerman, the density of low-lying zeros of L-functions in a family weighted by central L-values has been studied by Knightly and Reno, the speaker, and Fazzari. Moreover, the speaker suggested the "Weighted Density Conjecture" on the density of low-lying zeros of L-functions weighted by general L-values. In this talk, we give evidence of the speaker's conjecture for a family of Dirichlet L-functions. This is a joint work with Ade Irma Suriajaya (Kyushu University).

14:10 – 14:50 **Masatoshi Suzuki (Tokyo Institute of Technology)**

"On the screw function of the Riemann zeta function"

The notion of screw functions was introduced in functional analysis in the 1970s to study positive-definite functions or kernels. In this talk, we define the screw function associated with the Riemann zeta function and describe its properties. Typical results are several equivalent conditions for the Riemann hypothesis in terms of the screw function including an analog of the so-called Weil's positivity criterion or Yoshida's non-degenerate criterion.

15:10 – 15:40 **Haruki Ide (Keio University)**

"Algebraic independence of the values and the derivatives of certain power series, infinite products, and Lambert type series"

Let $a_1, \dots, a_r$ be algebraic numbers with $0 < |a_i| < 1$ $(1 \le i \le r)$ and let $\{R_k\}_{k\ge 0}$ be a certain linear recurrence including Fibonacci sequence. We consider the power series $F_{i,m}(x) := \sum_{k=0}^{\infty} a_i^{mR_k} x^k$ $(1 \le i \le r,\ m \ge 1)$ and the infinite products $G_i(y) := \prod_{k=0}^{\infty}(1-a_i^{R_k}y)$ $(1 \le i \le r)$. Tanaka proved that, for each i $(1 \le i \le r)$, the infinite set $S_i := \{F_{i,m}^{(l)}(\alpha) \mid \alpha \in \overline{\mathbb{Q}}^{\times},\ l \ge 0,\ m \ge 1\}$ is algebraically independent and so is the infinite set $T_i := \{G_i^{(m)}(\beta) \mid \beta \in \overline{\mathbb{Q}}^{\times} \setminus \{a_i^{-R_k}\}_{k\ge 0},\ m \ge 0\}$. Considering the r algebraic numbers $a_1, \dots, a_r$ simultaneously, we present in this talk a necessary and sufficient condition on the $a_1, \dots, a_r$ for the union $S_1 \cup \cdots \cup S_r \cup T_1 \cup \cdots \cup T_r$ to be algebraically independent. This result is obtained as a corollary to a more general theorem which deals with the algebraic independence of the values and the partial derivatives of certain Lambert type series of two variables.

16:00 – 16:40 **Wataru Takeda (Tokyo University of Science)**

"Topological properties and algebraic independence of sets of prime-representing constants"
(joint work with Kota Saito)

For a sequence $(c_k)_{k\in\mathbb{N}}$ of positive integers, we consider the set $\mathcal{W}(c_k)$ of all $A > 1$ such that the integer parts of $A^{c_1 \cdots c_k}$ are prime numbers for all $k \in \mathbb{N}$. In 1947, Mills first considered the set $\mathcal{W}(3)$ and showed $\mathcal{W}(3) \neq \emptyset$. For a general sequence $(c_k)_{k\in\mathbb{N}}$, Alkauskas and Dubickas constructed a transcendental number in $\mathcal{W}(c_k)$ if a real sequence $(c_k)_{k\in\mathbb{N}}$ satisfies some conditions in 2004. In this talk, we present an improvement on their results. We give an algebraically independent and infinitely countable subset of $\mathcal{W}(c_k)$ if $(c_k)_{k\in\mathbb{N}}$ is rapidly increasing. As a corollary, we disclose that the minimum of $\mathcal{W}(c_k)$ is transcendental if c_k goes to infinity. This

research is joint work with Kota Saito (University of Tsukuba).

October 14 (Fri)

9:30 – 10:30 **Kohji Matsumoto (Nagoya University)**

"On the value-distribution of the logarithms of symmetric power L-functions in the level aspect"
(joint work with Philippe Lebacque, Masahiro Mine and Yumiko Umegaki)

We report new results on the value-distribution of the logarithm of symmetric power L-functions associated with new forms of prime-power level, in view of the level aspect. Under certain analytical conditions, we prove a limit theorem expressed by an integral involving M-functions, in the case of symmetric square L-functions. We further report that the same type of result also holds in general case for some special form of test functions, and mention a certain parity property of M-functions.

10:50 – 11:30 **Hirofumi Nagoshi[Z] (Gunma University)**

"Joint probability distribution and its density function for values of the logarithms of the Riemann zeta-function and related functions"

Bohr and Jessen obtained a probabilistic limit theorem for values of the logarithm of the Riemann zeta-function $\zeta(s)$, and showed that the limit distribution has a density function. Similar or related results have been established by many researchers including Ihara, Matsumoto, Mine and Umegaki. In this talk, we will discuss joint probability distribution for values of the logarithms of several L-functions $L_1(s), \dots, L_N(s)$ satisfying suitable conditions, and, in particular, show that the limit distribution has a joint density function. A crucial ingredient in the proof is the Selberg orthogonality for those L-functions.

Kershner and Wintner obtained an analogue of the Bohr-Jessen limit theorem for values of the logarithmic derivative $\zeta'(s)/\zeta(s)$ of $\zeta(s)$. We will also discuss joint probability distribution and its joint density function for values of the functions $\log \zeta(s)$ and $\zeta'(s)/\zeta(s)$.

13:10 – 13:50 **Makoto Kawashima (Nihon University)**

"On linear independence of values of hypergeometric functions"
(joint work with Sinnou David and Noriko Hirata-Kohno)

In this talk, we show a new general linear independence criterion related to values of G-functions, including the linear independence of values at algebraic points of contiguous hypergeometric functions, which is not known before. The statement is as follows. Let K be any algebraic number field and v be a place of K. Let $r \in \mathbb{Z}$ with $r \geq 2$. Consider $a_1, \dots, a_r, b_1, \dots, b_{r-1} \in \mathbb{Q} \setminus \{0\}$ not being negative integers. Assume neither a_k nor $a_k + 1 - b_j$ be strictly positive integers $(1 \leq k \leq r, 1 \leq j \leq r-1)$. Let $\alpha_1, \dots, \alpha_m \in K \setminus \{0\}$ with $\alpha_1, \dots, \alpha_m$ pairwise distinct. By choosing sufficiently large $\beta \in \mathbb{Z}$ depending on K and v such that the points $\alpha_1/\beta, \dots, \alpha_m/\beta$ are closed enough to the origin, we prove that the $rm+1$ numbers :

$$ {}_rF_{r-1}\left(\begin{matrix} a_1, \dots, a_r \\ b_1, \dots, b_{r-1} \end{matrix} \middle| \frac{\alpha_i}{\beta}\right) \ , \quad {}_rF_{r-1}\left(\begin{matrix} a_1+1, \dots, \dots, \dots, a_r+1 \\ b_1+1, \dots, b_{r-s}+1, b_{r-s+1}, \dots, b_{r-1} \end{matrix} \middle| \frac{\alpha_i}{\beta}\right) $$
$$ (1 \leq i \leq m, 1 \leq s \leq r-1) $$

and 1 are linearly independent over K. This is a joint work with S. David and N. Hirata-Kohno.

14:10 – 14:40 **Takuki Tomita (Keio University)**

"On the series expression of the logarithmic derivative of an absolute zeta function and its

[Z] Talk by Zoom online

absolute Euler product"
(joint work with Yoshinosuke Hirakawa)

In 2010's, using zeta regularization, Kurokawa defined an absolute zeta functions from a given function. If the given function is a polynomial with integral coefficients, he also identified the absolute zeta function with a certain infinite product, which he called the absolute Euler product. In this talk, we focus on absolute zeta functions obtained from analytic functions over $[1, \infty)$ satisfying some conditions. Firstly, we show that the logarithmic derivative of such an absolute zeta function can be identified with the generating function of the iterative Euler derivatives of the given function. Secondly, we extend Kurokawa's result on the absolute Euler product in the case when the given function is a $\mathbb{C}$-linear combination of t^ρ's ($\rho \in \mathbb{C}$). Additionally, under mild conditions, we also determine the region of absolute convergence for this absolute Euler product. This talk is based on a joint work with Yoshinosuke Hirakawa, Tokyo University of Science.

15:00 – 15:30 **Yuichiro Toma (Nagoya University)**
"Analytic properties and mean values of several double zeta-functions"

In the case of the Riemann zeta function, the study of its mean value order on the critical line is considered an approach to elucidate the Lindelöf conjecture. Recently, analytical properties of the multiple zeta functions have been investigated by considering the order of their mean values. In this talk, we present results on the asymptotic behavior of the mean values of two types of double zeta functions, namely, Mordell-Tornheim and Apostol-Vu types.

15:50 – 16:30 **Ade Irma Suriajaya (Kyushu University)**
"Zeros of derivatives of L-functions in the Selberg class on the left-half plane and the left-half of the critical strip"
(joint work with Sneha Chaubey and Suraj Singh Khurana)

Speiser in 1935 showed that the Riemann hypothesis is equivalent to the first derivative of the Riemann zeta function having no non-real zeros to the left of the critical line. This was reproved by Levinson and Montgomery in 1974 in a quantitative form. Analogous results have been proven for other L-functions, including Dirichlet L-functions associated to primitive characters, by Akatsuka and myself. We are interested in extending these results to the Selberg class. This has been studied by Šleževičienė, also extended by Garunkštis and Šimėnas to L-functions in the extended Selberg class which includes L-functions not satisfying the Riemann Hypothesis, but only in the T-aspect. We are interested in obtaining more precise estimates showing explicit dependence on other parameters. For that purpose, we also need more explicit results on zero free regions and trivial zeros. In this talk, I would like to present these initial results which have been summarized in our recent paper. This is an ongoing project with Sneha Chaubey (IIIT-Delhi) and Suraj Singh Khurana (Indian Institute of Technology Kanpur).