Calculations for Broué’s abelian defect group conjecture
ブルーの可換不足群予想の計算

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This is a joint work with Naoko Kunugi and Katsushi Waki, and a detailed version of a result presented here is in [6].

It has been conjectured by Michel Broué that a block algebra of a finite group should be derived (Rickard) equivalent to a block algebra of the normalizer of a common defect group which correspond each other via the Brauer correspondence provided the defect group is abelian, see [2, 6.2.Question]. This is known as Broué’s Abelian Defect Group Conjecture, (ADGC) for short. We have been continuing a project on Broué’s ADGC for a specific defect group, say the elementary abelian group of order nine, see [3], [4], [5]. Our main result here is the following:

**Theorem** (Koshitani-Kunugi-Waki, 2005). Let \( G \) be the Janko simple group \( J_4 \), and let \((O, K, k)\) be a splitting 3-modular system for all subgroups of \( G \), namely, \( O \) is a complete discrete valuation ring of rank one such that \( K \) is the quotient field of \( O \) with \( \text{char}(K) = 0 \) and such that \( k \) is the residue field of \( O \), namely \( k = O/\text{rad}(O) \), with \( \text{char}(k) = 3 \), and \( K \) and \( k \) are both splitting fields for all subgroups of \( G \). Let \( A \) be a unique block algebra of \( OG \) whose defect group \( P \) is elementary abelian of order 9, and let \( B \) be the Brauer correspondent of \( A \) in \( OH \) where \( H = N_G(P) \). Then, \( A \) and \( B \) are derived (Rickard) equivalent. In fact, even stronger fact is proved, namely, \( A \) and \( B \) are splendidly derived (Rickard) equivalent, see [9] and [10].
Remark. In our proof results in papers of Okuyama [7] and [8] are important.

Corollary. It turns out that Broué’s ADGC holds for any prime $p$ and any block algebra of $G$. This means that Broué’s ADGC is settled for all primes and all block algebras of $J_4$.

Proof. This follows immediately from Theorem and [1, Lemma 5.1].

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References